MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1 2022/2023 (TERM ID 2210)

BNA3014 – NUMERICAL ANALYSIS

(All sections / Groups)

16 FEBRUARY 2023
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **4 pages** including formulae sheets with **4 questions** only.
2. Attempt **ALL** questions and write your answers in the Answer Booklet provided.
3. Students are allowed to use non-programmable scientific calculators that are permitted to be used in the examination.

Question 1 (25 Marks)

(a) Consider the following function,

$$f(x) = \sqrt{x^4 + 4} - 2$$

i. Suggest a function $g(x)$ that will avoid loss of significance error of $f(x)$. [4 marks]

ii. Compute $f(0.1)$ and $g(0.1)$ by using three-significant digits and rounding. [6 marks]

(b) Given the Maclaurin series for logarithm is

$$e^y = \sum_{n=0}^{\infty} \frac{y^n}{n!}$$

i. Approximate $f(y) = e^{2y} - e^{-y}$ that will avoid loss of significance when y approaches zero, with the truncated 5 terms of Maclaurin series. [6 marks]

ii. Integrate the series for $f(y)$ in part (i) in the interval of (0,1). [3 marks]

(c) Rewrite the given polynomial, $p(x)$ in nested form, $r(x)$. Use three-digits and rounding to evaluate both $p(x)$ and $r(x)$ at x =2.15.

$$p(x) = x^4 + 2x^3 - 13x^2 - 14x + 4$$

[6 marks]

Question 2 (25 Marks)

(a) For the following function

$$f(x) = x^4 - 5x^3 + 9x + 3 = 0$$

with starting point is the midpoint of the interval [4,6],

i. determine the number of iterations, n needed such that

$$|p_n - p_{n-1}| < 7 \times 10^{-2}$$ [2 marks]

ii. find a root, p_n of the function, $f(x)$ by using Newton's Method at n as obtained in (i).

Note: Write your answer correct to four-decimal places. [12 marks]

(b) Use Gaussian Elimination with partial pivoting to solve the following system of linear equations.

$$\begin{aligned} 2x_1 - 3x_2 + 2x_3 &= 5 \\ -4x_1 + 2x_2 - 6x_3 &= 14 \\ 2x_1 + 2x_2 + 4x_3 &= 8 \end{aligned}$$

[11 marks]

Continued .....

Question 3 (25 Marks)

(a) For the following given table:

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	9.9833	4.9667	3.2836	2.4339	1.9177

i. Construct a Newton's divided difference table. [7 marks]

ii. Using table in part (i), write the Newton polynomial $P_3(x)$ and approximate it for x=0.25. [4 marks]

(b) For the following data:

x	0	0.5	1.0	1.5	2.0	2.5
y	-0.4326	-0.1656	3.1253	4.7877	4.8535	8.6909

Obtain a model that represents this data by using the method of least square. Fit the model equation of the form $y = a + bx$. [14 marks]

Question 4 (25 Marks)

(a) Use the Trapezoidal Rule to find the approximation of the integral $\int_1^3 \sqrt[3]{x + x^6}\, dx$

by taking four equal subintervals.

[13 marks]

(b) Apply the Runge-Kutta method of order 2 to solve the following initial value problem

$$\frac{dy}{dx} + 2y = 1.3e^{-x}, \qquad y(0) = 0.5$$

for the range $0 \leq x \leq 1$ and $h = 0.5$.

[12 marks]

End of Page

A. ERROR ANALYSIS

Taylor Polynomial :

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(a)(x-a)^k$$

Maclaurin Series :

(i) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ (ii) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (iii) $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

(iv) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ (v) $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n!}$

B. SOLUTION OF NONLINEAR EQUATIONS

Newton-Raphson Method: $p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$

Secant Method: $p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$

C. SOLUTION OF LINEAR SYSTEMS

Gauss-Jacobi Method : $x_i^k = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k-1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k-1} \right]$

Gauss-Seidel Method: $x_i^k = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{k-1} - \sum_{j=i+1}^{n} a_{ij} x_j^{k-1} \right]$;

where $x_j^{k-1} = x_j^k$ based on the latest updates

D. INTERPOLATION AND POLYNOMIAL APPROXIMATIONS

Lagrange Polynomial:

$$P_n(x) = \sum_{i=0}^{n} \frac{1}{k!} f(x_i) L_i(x)$$

Newton Divided Difference:

$$P_n(x) = a_0 + a_1(x - x_0) + \cdots .. + a_n(x - x_0)(x - x_1) \ldots (x - x_{n-1})$$

where

$$a_n = f[x_0, x_1, \ldots . x_n]$$

E. NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

Least square fitting:

(i) Linear : $y = a + bx \quad ; \sum y = na + b\sum x \quad ; \sum xy = a\Sigma x + b\sum x^2$

(ii) Exponential: $y = ae^{bx} \quad ; \sum z = nA + b\sum x \quad ; \sum xz = A\sum x + b\sum x^2$

Trapezoidal Rule:

$$A = \frac{h}{2}\left[\left(f(x_1) + f(x_{n+1})\right) + 2(f(x_2) + f(x_3) + \cdots + f(x_n))\right]$$

Simpson's 1/3 Rule:

$$A = \frac{h}{3}\left[\left(f(x_1) + f(x_{2n+1})\right) + 4(f(x_2) + f(x_4) + \cdots f(x_{2n})) + 2(f(x_3) + f(x_5) + \cdots + f(x_{2n-1}))\right]$$

F. SOLUTION OF DIFFERENTIAL EQUATIONS

Euler Method:

$$y_{n+1} = y_n + hf(x_n, y_n) \quad \text{where } n = 0,1,2,3,\cdots$$

Runge-Kutta second order Method:

$$k_1 = f(x_n, \quad y_n)$$

$$k_2 = f(x_n + h, \quad y_n + hk_1)$$

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2)$$